

MATH 141: Quiz 3

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Find the derivatives of the following functions:

1. $f(x) = \sin(x) \cos(x)$ *Use product rule.*
factor · factor

$$\begin{aligned} f'(x) &= \cos(x) \cdot \frac{d}{dx} [\sin(x)] + \sin(x) \cdot \frac{d}{dx} [\cos(x)] \\ &= \cos(x) \cdot \cos(x) + \sin(x) \cdot (-\sin(x)) \\ &= \boxed{\cos^2(x) - \sin^2(x)} \end{aligned}$$

2. $g(x) = \frac{3x^2 - 1}{4x^3 + 2x}$ *use quotient rule.*
top
bottom

$$\begin{aligned} g'(x) &= \frac{(4x^3 + 2x) \cdot \frac{d}{dx} [3x^2 - 1] - (3x^2 - 1) \cdot \frac{d}{dx} [4x^3 + 2x]}{(4x^3 + 2x)^2} \\ &= \frac{(4x^3 + 2x) \cdot 6x - (3x^2 - 1)(12x^2 + 2)}{(4x^3 + 2x)^2} \quad \text{subtracting 4 terms, don't forget} \\ &= \frac{24x^4 + 12x^2 - (36x^4 + 6x^2 - 12x^2 - 2)}{(4x^3 + 2x)^2} \\ &= \frac{24x^4 + 12x^2 - 36x^4 - 6x^2 + 12x^2 + 2}{(4x^3 + 2x)^2} = \frac{-12x^4 + 18x^2 + 2}{(4x^3 + 2x)^2} = \frac{-2(6x^4 - 9x^2 - 1)}{(4x^3 + 2x)^2} \end{aligned}$$

3. $h(x) = \sqrt[3]{\tan(x^2)} = (\tan(x^2))^{\frac{1}{3}}$ *use chain rule two times.*

$$\begin{aligned} h'(x) &= \frac{d}{dx} \left[(\tan(x^2))^{\frac{1}{3}} \right] \\ &= \frac{1}{3} (\tan(x^2))^{\frac{1}{3}-1} \cdot \frac{d}{dx} [\tan(x^2)] \\ &= \frac{1}{3} (\tan(x^2))^{-\frac{2}{3}} \cdot \sec^2(x^2) \cdot \frac{d}{dx} [x^2] \\ &= \frac{2x \cdot \sec^2(x^2)}{3 \sqrt[3]{\tan^2(x^2)}} \end{aligned}$$

4. $f(x) = \underbrace{(2x-1)^4}_{\text{factor}} \underbrace{(3x+2)^3}_{\text{factor}}$ *use product rule.*

chain (general power rule) *chain (general power rule)*

$$\begin{aligned} f'(x) &= (3x+2)^3 \cdot \frac{d}{dx} [(2x-1)^4] + (2x-1)^4 \cdot \frac{d}{dx} [(3x+2)^3] \\ &= (3x+2)^3 \cdot 4(2x-1)^3 \cdot \frac{d}{dx} [2x-1] + (2x-1)^4 \cdot 3(3x+2)^2 \cdot \frac{d}{dx} [3x+2] \end{aligned}$$

$$\begin{aligned} &= \underbrace{(3x+2)^3 \cdot 4(2x-1)^3 \cdot 2}_{\text{two term factorization problem. Use GCF first.}} + \underbrace{3 \cdot (2x-1)^4 (3x+2)^2 \cdot 3} \\ &= (3x+2)^2 (2x-1)^3 \left(\underbrace{8(3x+2)}_{\text{expand because like terms.}} + \underbrace{9(2x-1)}_{\text{expand because like terms.}} \right) \end{aligned}$$

$$= (3x+2)^2 (2x-1)^3 (24x+16+18x-9)$$

$$= \boxed{(3x+2)^2 (2x-1)^3 (42x+7)}$$